@whisto_maths

Trigonometry

What do I need to be able to do?

By the end of this unit you should be able to:

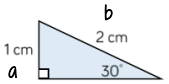
- Work fluently with hypotenuse, opposite and adjacent sides
- Use the tan, sine and cosine ratio to find missing side lengths
- Use the tan, sine and cosine ratio to find missing angles
- Calculate sides using Pythagoras' Theorem

Keywords

Enlarge: to make a shape bigger (or smaller) by a given multiplier (scale factor)
Scale Factor: the multiplier of enlargement
Constant: a value that remains the same
Cosine ratio: the ratio of the length of the adjacent side to that of the hypotenuse. The sine of the complement.
Sine ratio: the ratio of the length of the opposite side to that of the hypotenuse.
Tangent ratio: the ratio of the length of the opposite side to that of the adjacent side.
Inverse: function that has the opposite effect.
Hypotenuse: longest side of a right-angled triangle. It is the side opposite the right-angle.

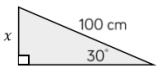
Ratio in right-angled triangles

When the angle is the same the ratio of sides a and b will also remain the same

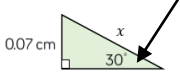


a : b
1 : 2

$\frac{a}{b} = \frac{1}{2}$



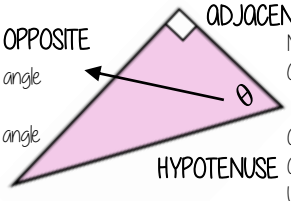
a : b
x : 100
50 : 100



a : b
0.07 : x
0.07 : 0.14

Hypotenuse, adjacent and opposite

ONLY right-angled triangles are labelled in this way



OPPOSITE
Always opposite an acute angle
Useful to label second
Position depend upon the angle in use for the question

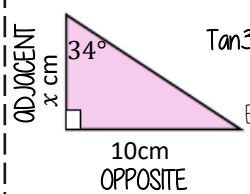
ADJACENT
Next to the angle in question
Often labelled last

HYPOTENUSE
Always the longest side
Always opposite the right angle
Useful to label this first

Tangent ratio: side lengths

$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}$

Substitute the values into the tangent formula



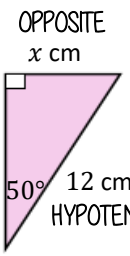
$\tan 34 = \frac{10}{x}$

Equations might need rearranging to solve

$x \times \tan 34 = 10$

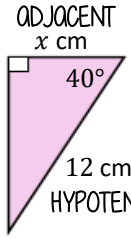
$x = \frac{10}{\tan 34} = 14.8\text{cm}$

Sin and Cos ratio: side lengths



$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse side}}$

NOTE
The $\sin(x)$ ratio is the same as the $\cos(90-x)$ ratio



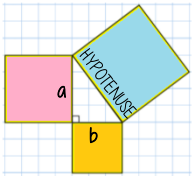
$\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse side}}$

Substitute the values into the ratio formula

Equations might need rearranging to solve

Pythagoras theorem R

$\text{Hypotenuse}^2 = a^2 + b^2$

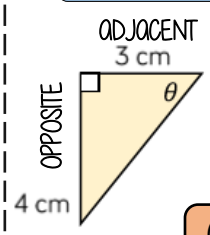


This is commutative – the square of the hypotenuse is equal to the sum of the squares of the two shorter sides

Places to look out for Pythagoras
- Perpendicular heights in isosceles triangles
- Diagonals on right angled shapes
- Distance between coordinates
- Any length made from a right angles

Sin, Cos, Tan: Angles

Inverse trigonometric functions



Label your triangle and choose your trigonometric ratio

Substitute values into the ratio formula

$\tan\theta = \frac{3}{4}$

$\theta = \tan^{-1}\frac{3}{4}$

$\theta = 36.9°$

$\theta = \tan^{-1}\frac{\text{opposite side}}{\text{adjacent side}}$

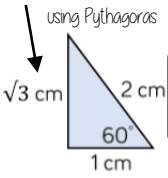
$\theta = \sin^{-1}\frac{\text{opposite side}}{\text{hypotenuse side}}$

$\theta = \cos^{-1}\frac{\text{adjacent side}}{\text{hypotenuse side}}$

Key angles

This side could be calculated using Pythagoras

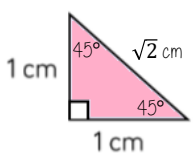
Because trig ratios remain the same for similar shapes you can generalise from the following statements



$\tan 30 = \frac{1}{\sqrt{3}}$ $\tan 60 = \sqrt{3}$

$\cos 30 = \frac{\sqrt{3}}{2}$ $\cos 60 = \frac{1}{2}$

$\sin 30 = \frac{1}{2}$ $\sin 60 = \frac{\sqrt{3}}{2}$



$\tan 45 = 1$

$\cos 45 = \frac{1}{\sqrt{2}}$

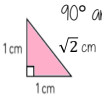
$\sin 45 = \frac{1}{\sqrt{2}}$

Key angles 0° and 90°

$\tan 0 = 0$

~~$\tan 90$~~

This value cannot be defined – it is impossible as you cannot have two 90° angles in a triangle



$\sin 0 = 0$ $\sin 90 = 1$

$\cos 0 = 1$ $\cos 90 = 0$